MONTRÉAL EXCHANGE

Bond Futures

Reference Manual

CGZ	Two-Year Government of Canada Bond Futures
CGF	Five-Year Government of Canada Bond Futures
CGB	Ten-Year Government of Canada Bond Futures
LGB	30-Year Government of Canada Bond Futures
OGZ	Options on Two-Year Government of Canada Bond Futures
OGF	Options on Five-Year Government of Canada Bond Futures
OGB	Options on Ten-Year Government of Canada Bond Futures

TMX

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The masculine form used in this document designates without discrimination both women and men.

Introduction

In September 1989, the Montréal Exchange (MX) launched the Ten-Year Government of Canada Bond Futures (CGB®). Since the introduction of its CGB contract, MX continued developing the Canadian yield curve by launching the Two-Year (CGZ®), Five-Year (CGF®) and 30-Year (LGB®) Government of Canada Bond Futures Accompanying the 10-year futures contracts are the options on CGBs (OGB®), adding more flexibility to managing interest rate risk. Through different applications of bond contracts, investors are able to both extract and preserve the value of capital pledged without term credit concerns.

The ten-year derivative product became popular with many bond dealers as it offers opportunities for portfolio enhancement and for use as a risk management tool. Asset managers have expanded their investment strategies to include the use of futures to adjust duration, hedge anticipated interest rate moves and have gone so far as to create investment funds dedicated solely to the total return of CGBs.

The objective of this document is to explain how Government of Canada bond futures are used. Its purpose is to familiarize the public with the flexibility offered by the various interest rate products representing Government of Canada bonds. MX offers pure price competition, via SOLA, allowing each participant equal access to the best available price.

The Government of Canada bond market

The Government of Canada first issued bonds to the public in the 1940s to subsidize the war effort. These "Victory Bonds" have since matured, and their principal repaid. However, financing the ongoing fiscal requirements of the Government of Canada has required continuous issuances of bonds. The Bank of Canada on behalf of the Government issues Government of Canada marketable securities (bonds) by auctioning their debt on prearranged auction dates to the highest bidders.

Bond auction policies change from government to government, including new types of issues and eliminating others. Currently, the Federal Department of Finance has established five different maturities of bond issuances and one issue of real return bonds per calendar quarter. The 2-, 3-, 5-, 10- and 30-year auction dates, the amounts, settlement dates and other details are announced quarterly. The Bank of Canada auctions these issues to primary distributors of Government of Canada marketable bonds.

Trading Futures contracts (CGZ, CGF, CGB, LGB): Mechanicals aspects

Who uses bond futures contracts?

Bond futures contracts are used for hedging (risk management), directional trading (income generation) and arbitrage (profit from market anomalies).

Hedging consists of operations that minimize or eliminate risk arising from the fluctuations of an underlying bond or any security having similarities with these bonds (i .e. yield, maturity). A buy position or a "long" position in an underlying bond or security can be covered by a sell position or "short" position in the futures. Conversely, a "short" position in the underlying bond can be covered by a "long" position in futures. The greater the correlation between the two, the better the hedge will be. Therefore, the loss in one market will be partially or possibly entirely offset by the gains in the other.

Directional traders aim to take profit from potential moves in the market. They look for trends in the market and position themselves accordingly. Most of the time they try to maximize their profits in the shortest period of time (intraday) but some hold their position for longer periods defined by the trend. These traders benefit from an appealing financial leverage where great profits can be obtained if they make a correct prediction of the trend but on the other hand, the losses from an incorrect prediction can be just as big. Therefore, they must exercise great discipline in their speculative trading.

Arbitrage operations are aimed at profiting from pricing anomalies in the market (i .e . underlying bond vs . the futures or options vs . the futures) . Price anomalies usually exist for a very short time; they are a result of pricing inefficiencies and are quickly corrected by the arbitrageur. To be effective, these trades must lock in an immediate profit, have no risk of incurring a loss and necessitate no net investment.

MX's Government of Canada bond futures will provide the following advantages:

- Effective tool to convert a floating rate loan to a fixed term loan.
- Efficient way to set the borrowing or investment rate for an anticipated cash flow or commitment.
- Cost-efficient way to manage the risk profile of:
    - swap books
    - mortgage loan portfolio
    - asset-liability management
- Excellent instrument to substitute the liquidity of illiquid securities, such as mortgages, certain corporate sectors, and private placements.
- Added efficiency in hedging Government of Canada bond holdings and Canadian long-term interest rate exposures.
- Added flexibility in trading interest rate differentials or yield spreads between Government of Canada bonds and government bonds of other major countries.
- Cost-efficient and simple way of "shorting" the Government of Canada bond market, enabling trading on falling or rising Government of Canada bond prices/yields.
- Effective tool for yield enhancement of Government of Canada bond portfolio.
- Efficient access at low cost to the Government of Canada bond market for all market participants in Canada and abroad.
- Facilitated sales and trading of Government of Canada bonds both in Canada and in the international markets.

Taking a position: short or long

A bond futures contract is an agreement traded on an exchange that obligates the contracting parties to buy or sell a fixed amount of bonds at a future date, but at a price agreed upon in advance. It is entered into by two different parties: the seller (the short) and the buyer (the long). Once a position has been taken in a futures contract, two alternatives are available. On one hand, the contract will be held until expiry, in which case the short will have to make delivery to the long, who will take delivery of an eligible bond[1] at a price established in advance. On the other hand, the contract may be closed by taking the opposite position in the same contract: short for the long, and long for the short.

Required margins

An initial margin is required from all approved participants and their clients. This good faith deposit ensures the financial position of both counterparts to a trade. Deposits are held at the central clearinghouse, the Canadian Derivatives Clearing Corporation (CDCC), and are marked to market on a daily basis. Various types of collateral can be deposited to meet the initial margin requirements including cash, government securities or similar highly liquid instruments. A client's minimum margin deposit is established through the use of a risk-based system and varies for speculators and hedgers. Full details concerning margin requirements can be obtained from the Montréal Exchange Regulatory Division' Futures Contracts Margin Rates page: reg.m-x.ca/en/regulatory/futures_margin

1. MX publishes the list of eligible or deliverable bonds known as the basket.

Position reporting threshold and position limits

The Bourse has a position reporting facility, which requires approved participants to supply details on positions over 250 futures contracts, referred to as Large Open Positions Reporting (LOPR). More details can be obtained at: reg.m-x.ca/en/regulatory/lopr.

Information on general position limits can be obtained from the Bourse as they are subject to periodic changes. For more details, refer to: reg.m-x.ca/en/regulatory/position_limits.

Cash settlements obey the following rules

	Interest Rates ▲	Interest Rates ▼
Futures price	Decrease	Increase
Long position (buyer)	Makes cash payment	Receives cash payment
Short position (seller)	Receives cash payment	Makes cash payment

When a futures contract is closed out before expiry, the total gain or loss is the cumulative amount of receipts or payments made on the daily cash settlements. For both the long and the short, this is equal to the sale price minus the purchase price.

$$\textbf{Profit / loss}_{\text{Long or short}} = \textbf{Sale price} - \textbf{Purchase price}$$

The delivery process

The following section details the delivery process for participants wishing to settle their contracts through physical delivery of Government of Canada bonds.

There are five important dates[2] in a bond futures contract life:

- The first notice day is one business day for CGZ, CGF and CGB contracts prior to the first business day of the delivery month and is the first day a short futures position (seller) may announce his intention to deliver the underlying instrument (cash bond) to the holder of the long futures position.
- A long futures position may receive delivery of the underlying cash bond at any time in the period between first delivery day (first business day of the delivery month) and last delivery day (last business day in the delivery month).
- The last trading day is seven business days prior to the last business day of the delivery month and is the final trading day of the futures contract. Thereafter, the delivery of all cash bonds will be set against the settlement price of the futures which is set at 1:00 p.m. on the last trading day.
- The last notice day is one business day for CGZ, CGF and CGB contracts prior to the last business day of delivery month and is the last day a short futures position (seller) may announce his intention to deliver the underlying instrument (cash bond) to the holder of a long futures position.

Example:

2. MX publishes expiration calendars for all derivative financial instruments offered. You can download a copy from the website at www.m-x.ca

The conversion factor system

As with other bond futures contracts, the CGB contract allows the seller to fulfill delivery obligations with one of the different bond issues which fit the delivery standards of each contract. The price of each deliverable bond will be calculated through the use of a conversion factor.

The conversion factor allows for the comparison of the deliverable Government of Canada bonds (with their varying coupons and maturities) on a common basis. It is calculated by determining the price at which a deliverable bond would have a semi-annual yield equal to the notional coupon.

The formula is as follows:

$$F = \frac{1/(1.03)^{d} \times [c/2 + c/0.06 \times (1 - 1/1.03^{n}) + 100/1.03^{n}] - c/2 \times (1 - d)}{100}$$

Where:

F = conversion factor;

c = coupon on $100 face amount;

n = number of half years from the first day of the futures delivery month to the final maturity date of the bond;

d = fractional part of n determined (after rounding down to the nearest whole three-month period[3]) as the number of whole three-month periods divided by six months, i.e. 0.0 or 0.5.

The notional coupon of the CGB contract is expressed as the parameters 0.06 and 1.03 (for compounding semi-annually) which flow throughout the formula.

Here is an example of deliverable Canadian government bonds and their respectable conversion factors for different months:

Deliverable Government of Canada Bond Issues and their Conversation Factors - CGB

as at August 9th, 2018

GOVERNMENT OF CANADA BONDS		Outstanding	September 2018	December 2018	March 2019	June 2019
Coupon	**Maturity**	C$ million	-	-	-	-
1%	**June 1, 2027**	15,000	0.6634	0.6708	0.6783	0.6860
2%	**June 1, 2028**	13,500	0.7079	0.7135	0.7191	0.7249
Total Outstanding Deliverable Bonds (C$ million)			28,500	28,500	28,500	28,500
Conversion factors computed with a yield equal to 6%						

3. The conversion factor for a deliverable bond is calculated in complete half-year periods from the first day of the futures contract month to the maturity date of the bond, with the number of excess months calculated in complete three-month periods (in the case of the CGB and LGB futures contracts) or in complete one-month periods (in the case of the CGZ and CGF futures contracts). Refer to contract specifications on pages 20-21 of the manual.

Pricing Bond Futures

Cheapest-to-deliver bond (CTD)

The cost of delivery of the bonds is influenced by both shifts in the yield curve and the fact that these same bonds trade more or less expensive than the yield curve would indicate. Therefore, the delivery price supplied by the conversion factor for each of the bonds in the delivery basket will tend to differ from its market price. This difference results in one of the bonds in the basket having less losses or greater gains than any of the other deliverable bonds. This bond is known as the cheapest-to-deliver (CTD).

The short will naturally choose to deliver the bond that is the least expensive or the cheapest-to-deliver. As a result, the futures price will tend to track the price of the CTD more closely than that of other bonds in the delivery basket . Determining the CTD calls for an understanding of the relationship between cash and futures markets.

Relationship between cash and futures prices

The conversion factor for a specific bond, during a specified period, remains constant in spite of changes in the price of the bond or the futures and represents a common point linking cash and futures markets. Thus, a simple operation with the conversion factor allows the futures price to be expressed in terms of cash prices or vice versa, in the following formulas:

Futures price expressed as a cash price:

$$\textbf{Cash equivalent price} = \textbf{Futures price} \times \textbf{Conversion factor}$$

Cash price expressed as a futures price:

$$\textbf{Futures equivalent price} = \frac{\textbf{Cash price}}{\textbf{Conversion factor}}$$

To simplify comparisons between the futures price and the different deliverable bond issues, we will use the futures equivalent price in the following discussions. The results will therefore not be in dollar values but can be transformed easily by multiplying the results by the conversion factor of the bond for which the calculation was done.

Theoretical futures price: cost of carry

For bond futures, in order to guarantee the delivery of a cash bond without any risk, the seller must purchase the bond at the moment the futures contract is established, and hold it until delivery. This involves the short- term cost of holding and financing the bonds until delivery, and the long-term yield received from the bonds during the same period. Therefore, prior to delivery, the futures equivalent price will have to be adjusted for the difference between the interest accrued from the coupon payments and the short-term financing rate (repo rate) for the period the position is held. The difference in the long-term yield and the short-term cost is known as cost of carry and is expressed as follows:

$$\textbf{Cost of carry} = \textbf{Coupon income} - \textbf{Financing cost}$$

More explicitly:

$$\textbf{Cost of carry} = (AD - AS) - (MV \times r \times t)$$

Where:

AS = accrued interest on the deliverable bond on the date the position is initially established;

AD = accrued interest on the deliverable at delivery, including coupons received since settlement and reinvested at the financing rate;

MV = market value (price + AS) of the deliverable bond;

r = financing rate (T-bill rate);

t = term (in years) from the date when the position is initially established to the delivery date.

In order to express the dollar value cost of carry of the CTD in terms of the futures price, it must be divided by the conversion factor of the CTD . We then obtain the adjusted cost of carry:

$$\textbf{Adjusted cost of carry}_{CTD} = \frac{\textbf{Cost of carry}_{CTD}}{\mathbf{f}_{CTD}}$$

Where:

f = conversion factor of the bond

Given the cost of carry, the theoretical equilibrium price of the bond futures relative to its underlying cash bond market price is the following:

$$\textbf{Theoretical futures price} = \textbf{Futures equivalent price}_{CTD} - \textbf{Cost of carry}_{CTD}\ \textbf{(adjusted)}$$

Or, alternately:

$$\textbf{Cost of carry}_{CTD}\ \textbf{(adjusted)} = \textbf{Futures equivalent price}_{CTD} - \textbf{Theoretical futures price}_{CTD}$$

The basis

Although the price of a futures contract will closely track the futures equivalent price of the CTD, a difference will be observed between the two. This difference is known as the basis and can be expressed as follows:

$$\textbf{Basis}_{CTD} = \textbf{Futures equivalent price}_{CTD} - \textbf{Actual futures price}$$

The basis may also be expressed in terms of the cash bond, i.e. in dollar value:

$$\textbf{Basis}_{CTD} = \textbf{Cash bond price}_{CTD} - (\textbf{Actual futures price} \times \mathbf{f}_{CTD})$$

In theory, the basis should be equal to the cost of carry, as stated by the theoretical futures price described previously. Thus, in theory, the investor should be indifferent to holding the long cash or short futures position, as the cost and yield would be the same.

The cost of carry will depend on the orientation of the yield curve. In the event of an upwardly sloping (normal) yield curve, the bond holder receives more income than the cost of financing, resulting in positive carry. In the case of a downward sloping (inverted) yield curve, the bond holder receives less income from holding the bonds than the cost of financing, which results in negative carry.

Given the relationship linking the cost of carry to the futures prices, a normal yield curve will result in the futures price being below the cash bond price. Alternately, under an inverted yield curve, the futures price will exceed the cash bond price in order to make up for the net loss on the bonds.

The price of futures will converge towards the price of the CTD as we near the contract expiry. This results from the cost of carry shrinking as the period during which the bond must be held diminishes. At expiry the cost of carry is zero.

Important concepts about the basis:

- “Long the basis” is: Long the Government of Canada bond, short the futures contract.
- The position represents owning an asset from the bond settlement day to the futures delivery day. In the cash market, it is known as a reverse repurchase agreement (reverse repo), which receives interest at the implied repurchase (repo) rate. A market participant agrees to purchase the CTD bond now and resell it in the future. This is an exchange of cash for bonds.

- "Short the basis" is: Short the Government of Canada bond, long the futures contract. The position represents owning a liability from the bond settlement day to the futures delivery day. In the cash market, it is known as a repurchase agreement (repo), which pays interest at the implied repurchase (repo) rate. A market participant agrees to sell the CTD bond now and repurchase it in the future. This is an exchange of bonds for cash.

Delivery options

The theoretical price we previously described is actually a simplification of the pricing of contract. In fact, bond futures prices will generally be slightly below the results expected by the cost of carry of the CTD. This is because the short in the futures also holds an implied put option.

The seller of the futures holds two options:

a. Option on which bond to deliver;

b. Option to provide delivery notice between the closing of the markets, during which time the underlying bond price may change with no effect on the futures settlement price . This option is referred to as a "wildcard".

Identifying the CTD

The bond that has greatest implied repo rate is cheapest-to deliver.

Implied repo rate:

It is basically the effective rate earned by purchasing a cash bond and selling the futures, with the intention of delivering that particular bond in the future.

By using the actual futures price, the cash price of the bond, the coupon income and taking into account the accrued interest, we can determine that rate with the following formula:

$$\text{IRR} = \frac{(F \times f) + AD - MV}{MV \times t}$$

Where:

F = futures settlement price;

f = conversion factor;

AD = accrued interest on deliverable at delivery, including any coupons received since settlement and reinvested at the financing rate;

MV = market value of the deliverable bond (price + accrued interest);

t = term (in years) from the date on which the position is initially established to the delivery date.

Another way of identifying the CTD is this rule of thumb.

Rule of thumb

Government Yield to Maturity is:	**Cheapest to deliver will be the bond in the basket with:**
LOWER than the notional coupon of the futures contract. Futures is at a premium (>100).	the **highest coupon** and **shortest maturity** (Shortest duration).
HIGHER than the notional coupon of the futures contract. Futures is at a discount (<100).	the **lowest coupon** and **longest maturity** (Longest duration).

It is important to remember that this guideline is true most of the time but numerous scenarios exist where other bonds may be the cheapest to deliver. It is especially true when the underlying bond yield is near the notional coupon of the futures contract. Duration will be explained further on when we look at the use of bond futures.

Basis risk

The risk that the futures contract does not perfectly track the bond that is being hedged is known as basis Risk. Basis risk is usually greater for bonds other than the CTD. As we have seen, futures contracts will generally converge towards the price of the CTD, especially in the final weeks prior to expiration.

Actual data on the basis tends to indicate that the futures contract will trade sometimes cheaper, and sometimes richer, during the life of the contract. Consequently, even when adjusted for the delivery option, the basis will not always equal the adjusted cost of carry, i .e . The actual futures price is not always equal to the theoretical futures price. Therefore, hedgers using futures for terms that end on dates other than contract expiry will experience basis risk, even when hedging with the CTD. The loss due to the basis risk is, in fact, the same as the profit gained from arbitrage transactions.

An astute hedger will aim to enter a position in order to profit from shifts in the basis. If possible, he will go short the futures when it is trading expensive (rich) and go long the futures when it is trading cheap.

Here are some general situations that may increase basis risk:

- changes in the slope of the yield curve;
- changes in yield spreads;
- changes in credit ratings;
- Changes in the short-term rates used to evaluate the cost of carry.

Note that these need not actually occur. The futures price may react as strongly to market sentiments as it would to real changes for any one of these situations.

Evolution of the cheapest-to-deliver bond

In our study of the basis and comparison of the theoretical to the actual futures price, we considered that the cheapest-to-deliver bond remained unchanged. However, the CTD can change during the life of the contract. Users of bond futures must familiarize themselves with the effects of a change in the CTD on their investment positions.

Supposing an investor holds the CTD with the intention of delivering it into a short futures position. As seen, in this classic cash-and-carry situation the investor has locked in future profits. A change in CTD means that this is no longer the cheapest deliverable bond. The investor may benefit from this situation by selling their bonds, purchasing the new CTD and delivering it into the futures. In this way, the investor secures greater gains than originally expected.

Alternately, an investor may have originally hedged a bond that was not the CTD and later became the CTD. In this case, the investor will once again increase his returns simply due to the change in the cheapest-to-deliver bond.

Generally, the CTD may change as a result of various causes:

- changes in the overall level of interest rates in Canada;
- change in yield spreads;
- Issue of a new eligible bond that becomes the cheapest to deliver.

A foreseen change in the CTD will impact on the delivery option. The higher the expectation of a change in the CTD, the higher the value of the delivery option, and the cheaper the actual futures price relative to its theoretical price.

Using Government of Canada Bond Futures

The hedge ratio

A hedge is generally defined as a transaction that reduces risk, usually at the expense of potential reward. Bond futures contracts are ideally suited to reduce interest rate risk over a specific period of time. A government bond futures hedge is achieved through the purchase or sale of an offsetting futures position in order to protect a position against interest rate risk.

The most important aspect of hedging with bond futures is the hedge ratio (the hedge ratio is directly influenced by the variation of the basis), which answers the question: how many futures contracts should be bought or sold? Because futures contracts and the position to be hedged often display different patterns of variation over time, the number of contracts necessary to offset the loss will tend to differ for each position. The hedge ratio is a measure of the relative price sensitivities of the futures contract (cheapest) and the position to be hedged, and is used to determine the necessary number of futures contracts to hedge a position.

Because arbitrage opportunities between the cash and the futures markets are generally achieved through cash- and-carry transactions based on the CTD, the price of the futures will track, and converge towards, the CTD. The hedge ratio must therefore reflect the price sensitivity of the CTD, as represented by the following formula:

$$\textbf{Hedge ratio} = \textbf{Relative price sensitivity} \times \textbf{Conversion factor}_{CTD}$$

and

$$\textbf{Number of contracts} = \textbf{Hedge ratio} \times \left[\frac{\textbf{Face value of cash bonds}}{\textbf{Nominal value of bond futures}} \right]$$

There are four basic approaches used to determine the hedge ratio or the relative price sensitivities:

- conversion factors;
- relative price sensitivity or one basis point value (BPV);
- duration;
- Regression analysis (yield beta).

Conversion factor hedge ratio

The price of a bond futures contract, adjusted by the conversion factor of the CTD, varies in direct relation to fluctuations in the price of the CTD. Therefore, the CTD can be hedged by applying the conversion factor as the hedge ratio. Because the relative price sensitivity in this case is one, the hedge ratio is the following:

$$\textbf{Hedge ratio} = \textbf{Conversion factor}_{CTD}$$

Our formula for the number of contracts thus becomes:

$$\textbf{Number of contracts} = \textbf{Conversion factor}_{CTD} \times \left[\frac{\textbf{Face value of cash bonds}}{\textbf{Nominal value of bond futures}} \right]$$

Conversion factors may also be used for bonds other than the CTD. However, as the futures price tracks and converges towards the price of the CTD, the hedge will only be effective if both bonds react in the same way to interest rate changes.

Relative price sensitivity, or basis point value (BPV)

When the bond to be hedged differs from the CTD in terms of coupon or maturity, relative price sensitivity and basis point value provide an estimate to the hedge ratio. The relative price sensitivity will use the ratio of price changes of the bonds for an expected yield change. The basis point value method will use the ratio of price changes of the bonds for a one basis point shift (0.01%) in the yield of the bonds. For both these methods, attention must be given in the case of large changes in interest rates, which result not only in price changes, but also in changes in the interest rate sensitivity of the bonds. The calculation of the BPV will be detailed in the following point on bond duration.

In this case the hedge ratio is obtained as follows:

$$\text{Hedge ratio}_{BPV} = \left(\frac{BPV_{\text{Hedged bond}}}{BPV_{CTD}} \right) \times \text{Conversion factor}_{CTD}$$

Our formula for the number of contracts thus becomes:

$$\text{Number of contracts} = \text{Hedge ratio} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

Duration hedge ratio

Duration

Duration is a measure of the life of a bond (in years) as it relates to change in its price. Technically, it is the present value weighted time to maturity of the cash flows of a fixed payment instrument, such as a Government
of Canada bond. The formula used to calculate the duration is the following:

$$\text{Macaulay duration} = \frac{\sum_{t=1}^{m} \frac{tC_t}{(1+y)^t}}{\sum_{t=1}^{m} \frac{C_t}{(1+y)^t}}$$

Where "t" is the period of each payment (that is 1, 2, 3... m), "$C_t/(1+y)^t$" is the present value of each payment (coupons and notional value), "y" is the yield-to-maturity of the bond and "m" is the number of periods . For example, a five-year bond that pays semi-annual coupons has m = 10.

The duration is a function of the bond's coupon rate, its term to maturity and its yield. The duration of a bond provides a measure of the sensitivity of the bond's price to a change in interest rates. The longer the duration of the bond, the greater will be the change in its prices as a result of a shift in interest rates. Prices of short-duration bonds have lower sensitivity to interest rate movements. An anticipation of higher interest rates leads to a greater demand for short-duration bonds, whereas anticipations for lower interest rates (increase in prices) lead to the acquisition of long-duration bonds.

Modified duration

The previous equation for duration must be modified slightly in order to be used as a measure of sensitivity. This is known as modified duration and is calculated as follows:

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{1 + y/p}$$

Where:

y = yield to maturity of the bond (in decimal form);

p = number of periods per year;

or:

y/p = periodic yield (in decimal form).

The modified duration can be used to determine the optimal hedge ratio. The equation for the hedge ratio obtained using the modified duration is the following:

$$\text{Hedge ratio}_{\text{Modified duration}} = \frac{\text{Cash price of bond}}{\text{Cash price of CTD}} \times \frac{\text{Modified duration of bond}}{\text{Modified duration of CTD}} \times \text{Conversion factor}_{\text{CTD}}$$

The number of futures contracts necessary for the hedge is calculated as follows:

$$\text{Number of futures contracts} = \text{Hedge ratio}_{\text{Modified duration}} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

In addition, coming back to our concept of BPV introduced in the previous point, once the modified duration is known, the price sensitivity to changes in interest rates is determined by the following:

$$\frac{\text{Change in price}}{\text{Price}} = -\text{Modified duration} \times \text{Change in interest rates}$$

The minus sign in the formula takes into account the fact that the price of bonds move inversely to interest rates. Dollar duration is a measure of the dollar price change resulting from a given change in interest rates, and is obtained by multiplying the result of the above equation by the price of the bond. Furthermore, by multiplying the above equation by the price of the bond and by a 0.01% change in interest rates, one obtains the basis point value (BPV) of the bond.

$$\text{BPV} = -\text{Modified duration} \times \text{Bond price} \times 0.01\% \text{ change in interest rates}$$

Regression analysis: correlation coefficient and yield beta (β)

The methods used above to calculate the hedge ratio are based on the theoretical (duration and BPV) and conventional (conversion factors) considerations. Because theory and conventions fail to capture all the aspects of market price evolution, the hedge ratio may be adjusted by comparing actual historical market data on the price or yield evolution of the CTD and the instrument or position to be hedged. Through the application of statistical regression techniques, we may evaluate either the correlation coefficient or the yield, both of which may be used to obtain a more accurate hedge ratio.

The correlation coefficient may be used directly in the following manner:

$$\text{Hedge ratio} = \text{Correlation coefficient} \times \text{Conversion factor}_{CTD}$$

The yield beta is used to adjust the hedge ratio of a duration or basis point value (BPV) weighted hedge as follows:

$$\text{Hedge ratio} = \frac{\text{Cash price of bond}}{\text{Cash price of CTD}} \times \frac{\text{Modified duration of bond}}{\text{Modified duration of CTD}} \times \text{Yield beta} \times \text{Conversion factor}_{CTD}$$

Which is the same as:

$$\text{Hedge ratio}_{BPV} = \frac{\text{BPV}_{\text{Hedged bond}}}{\text{BPV}_{CTD}} \times \text{Yield beta} \times \text{Conversion factor}_{CTD}$$

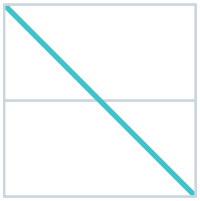
Options on Government of Canada Bond Futures (OGZ, OGF, OGB)

OGZ, OGF and OGB are options on Government of Canada bond futures contracts, which represent the right, but not the obligation, to buy (call) or sell (put) the underlying futures contract at a specified price (strike) in a specified amount of time (expiry). Options on Government of Canada bond futures are an added risk management feature of the underlying futures contract, enabling market participants to modify a portfolio's market risk to increase profit potential or reduce losses.

Price action of the underlying

As noted previously, Government of Canada bond futures move as a function of the cheapest-to-deliver bond. The risk position of futures is straightforward in comparison to options. Futures have two intrinsic measures. The sensitivity of upward and downward movement in the underlying futures contract is referred to as its delta. The delta of Government of Canada bond futures is $10 per tick (0.01) per contract. The second measure of sensitivity is a relative measure of the futures to the cheapest-to-deliver bond, which is the implied cost of carry or the futures basis.

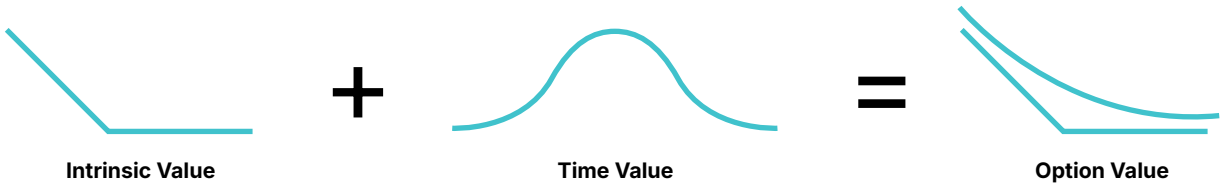
Long futures position benefits from price increases in the underlying, while losing value as the underlying decreases in price.



Short futures position benefits from price decreases in the underlying, while losing value as the underlying increases in price.

Options

Option buyers pay premium for the right of the option. This premium is made up of two parts, the intrinsic value and the time value. The intrinsic value of an option is the difference between the strike price and the market price. Upon expiry, the value of an option will be the maximum of 0 or the intrinsic value. The time value is a function of the cost of carry of the underlying futures contract, the time to expiry of the option, and the estimated market volatility.



Time value

Time value is assumed to be normally distributed around a mean (market price). It can be decomposed into three parts: the time to expiry, the cost of carry and the market volatility. Both the time to expiry and the fixed cost of carry are known (the basis of the underlying futures contract). The third variable, set by the marketplace, is the magnitude of the time value (premium), which is the price volatility of the underlying.

Volatility

Price movements (or returns) in the market can be measured against their movement away from a mean (average). A statistical analysis can be as rudimentary as a single linear regression analysis of past price action to far more complex geometrical translations of price trends in certain periods in time. Regardless of how models compare past price movement or historical volatility, pricing of options must encompass future volatility. This is why options may trade at a different volatility (market volatility) rate than the implied historical volatility.

Volatility smile

Option pricing models, such as Black-Scholes, assume that all options of a given maturity should be priced with the same level of implied volatility. This however is not what we observe in financial markets. The volatility smile is the pattern in which in- and out-of-the-money options are observed to have higher implied volatilities than at-the-money options. When implied volatility is plotted against the strike price, the resulting graph is typically valley-shaped. This volatility smile indicates that market participants believe extreme events have a greater probability of occurring than the normal distribution imply. Generally, a normal distribution implies a 1% chance of a three-standard-deviation move in the price of the underlying.

Intrinsic value

The intrinsic value of an option is the value that could be obtained if the holder decided to exercise the option immediately. It can thought of as the value of the underlying security that is built into the price of the option. In effect it is the dollar amount by which the option is in-the-money. Out-of-the-money and at-the-money options do not have any intrinsic value.

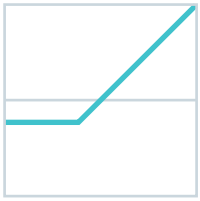
Let's consider a 135 OGB call option at a cost of $1.40. The premium paid for the option is the sum of the intrinsic value and time value. When time value goes to zero (expiry), the return on the call option strategy can be evaluated by comparing the call payoff to the price of the option. To make a profit on the purchase of the option, the market price of the CGB must be higher than the strike by the cost of the option.

Strike	+	Price of option	=	Break-even level
$135.00	+	$1.40	=	$136.40

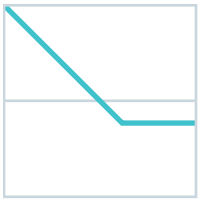
The break-even level on the 135 OGB calls is 136.40. This is the point that the return on the option position equals the investment in the option. The maximum loss is $1.40. The maximum gain is theoretically unlimited, as the price of the underlying can rise infinitely (though this outcome is not very likely).

MONEYNESS	Intrinsic price	Calls	Puts	Absolute delta[4]
In-the-money	Intrinsic positive	Strike < Market	Strike > Market	> 50%
At-the-money	Intrinsic near 0	Nearest strike to market	Nearest strike to market	50%
Out-of- the-money	Intrinsic 0	Strike > Market	Strike < Market	< 50%

Calls and puts



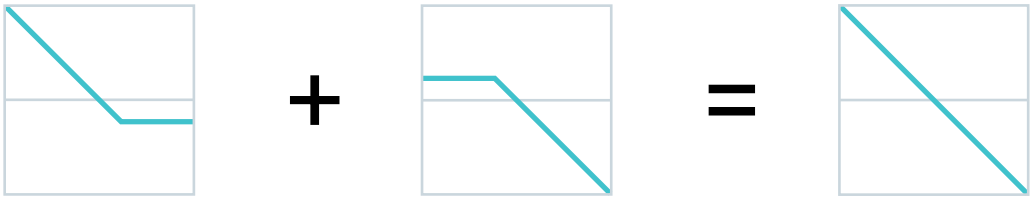
Call buyers have the right, but not the obligation, to buy a security at a fixed price in the future. They participate in the price appreciation of a security. If the price of the underlying increases, the price of the call will increase at an increasing rate (price convexity). Similarly, a decrease in the price of the underlying will decrease the value of the option at a decreasing rate . The maximum downside of a call option buyer is the premium. The maximum upside is hypothetically unlimited.



Put buyers have the right, but not the obligation, to sell a security at a fixed price in the future. They participate in the price depreciation of a security. If the price of the underlying decreases, the price of the put will increase at an increasing rate. Similarly, an increase in the price of the underlying will decrease the value of the option at a Decreasing rate. The maximum downside of a put option buyer is the premium. The maximum upside is limited to the price of the security going to zero.

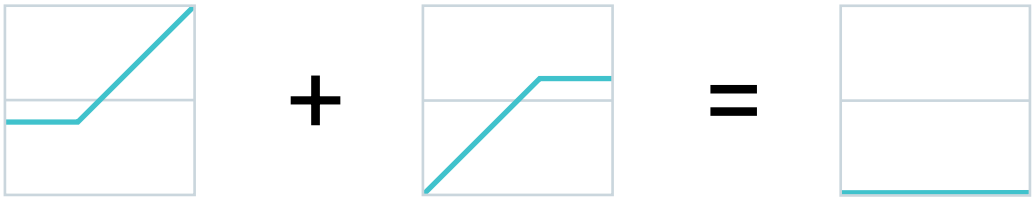
Put-call parity

Combining a long put position and a short call position is theoretically equivalent to a short futures position. Conversely, a long call position combined with a short put position is theoretically equal to a long futures position. When buying one option and selling another at the same strike, the time premium is both bought and sold. The remaining position is the expiry (intrinsic) payoff. The following diagrams detail this:



Long Put **Short Call** **Short Futures**

This parity gives a market participant a great number of possible trading and hedging strategies.



Long Call **Short Put** **Long Futures**

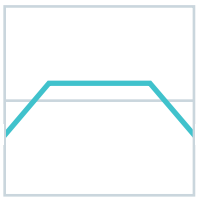
4. See Section Risk analysis of options.

Long Straddle

Short Straddle

A straddle is a combination of a call and a put struck at the same price. A long straddle is a purchase of a call and a put, a short straddle is a sale of a call and a put.

Long Strangle



Short Strangle

A strangle is a combination of an out-of-the-money call with an out-of-the-money put. A long strangle is a purchase of a call and a put, a short strangle is a sale of a call and a put.

Risk analysis of options

The 5 Greeks

The pricing of an option, as stated above, is simply the sum of the intrinsic value and the time value. The variables that affect the price of the option are therefore the moneyness of the option (the level of the underlying relative to the strike price), the term to maturity, the implied volatility and interest rates. The Black-Scholes formula provides the fundamental link between the option price and the above variables, but even more importantly, the Black-Scholes can be used to derive the sensitivity of the option price to changes in the levels of these different variables. By simply taking the partial derivative of the pricing formula with respect to each variable we can derive the risk parameters commonly known as the Greeks.

Delta: is the change in the options price with respect to the change in the price of the underlying. Typically, this is stated as a percentage amount of the underlying asset. Both a long call and a short put position will have positive deltas. Conversely, a long put and a short call position will have negative deltas. The absolute value delta of an at-the-money option is 50%, out-of-the-money < 50% and in-the-money > 50% .

Gamma: is the change in the delta with respect to a change in the underlying asset. Gamma is the second derivative of the price of an option with respect to the price of the underlying. Gamma typically grows larger for at-the-money options as the time to expiry draws closer.

Vega (Kappa): is the change in an option price with respect to a change in the implied volatility. Usually, this is stated in a one percent change in the volatility. The owner of an option contract is long volatility.

Theta: is the change in price of an option contract with respect to time. As the time to expiry draws closer, the time value of the option approaches zero. A long option position will lose its time value and thus option value. Institutional option traders further break theta into the two parts: daily accruals and the change in the value of the yield curve with respect to time.

Rho: is the change in the price of an option with respect to the risk-free rate of borrowing, or in the case of the CGBs the repo rate (represented by the basis of cash to futures). While this rate is integral to the pricing of the CGBs relative to the cash markets, the option price sensitivity is usually very low. Some multinational financial institutions break rho into financing risk (basis) and the cross-currency exposure of their reporting versus home currency.

Delivery

On the day of expiry, the option holder may choose to make or receive delivery, depending whether it is a call or put, for the underlying futures contract at the strike of the option. Option holders contact their broker to notify whether they will be making or receiving delivery. The brokers then inform the clearing house of the intentions of the option holder; the clearing house then informs the short option positions of their assignments. Typically, options contracts are automatically assigned if the options are in-the-money.

Margin

Margining options is similar to margining futures. A long option position need only pay the premium of the option, as the premium is the maximum downside of that position. Strategies combining several options require the amount of margin that will encompass the maximum downside of the strategy. If the maximum downside of an option strategy is more representative of an open futures position (i .e . short option position), the position will be margined as a combination futures and option position.

Canadian Derivatives Clearing Corporation (CDCC)

Canadian Derivatives Clearing Corporation (CDCC), a wholly-owned subsidiary of the Montréal Exchange (MX), acts as the central clearing counterparty for exchange-traded derivative products in Canada and for a growing range of customized financial instruments. CDCC's role is to ensure the integrity and stability of the markets that it supports. The CDCC requires each member to maintain margin deposits with the clearinghouse in order to cover the market risk associated with each member's positions. The assessment of this risk is based on a set of well-defined criteria established by the clearinghouse. Margins are collected daily or more frequently during periods of market volatility.

Trading Strategies

Here is the list of the bond future main ten strategies:

- Futures invoice spread
- Cash-and-carry trade
- Hedging the CTD
- Bond portfolio duration adjustment
- Hedging open swap positions
- Cross hedging: Hedging a portfolio of Canada Mortgage Bonds (CMBs)
- Trading on the yield curve
- Credit spread
- Hedge anticipated new issue purchases
- Cross-Currency trades

These strategies have applications in corporate treasury and portfolio management, as well as the management of swaps books, mortgage loans, and asset-liability matching.

Specific examples could be found on the Montreal Exchange website:
www.m-x.ca/educ_guides_strat_en.php

APPENDIX 1

Government of Canada bond futures specifications

	CGZ (Two-Year)	CGF (Five-Year)
Underlying	C$100,000 nominal value of a Government of Canada bond with a 6% notional coupon.	C$100,000 nominal value of a Government of Canada bond with a 6% notional coupon.
Expiry cycle	March, June, September and December	
Price quotation	Per C$100 nominal value	
Minimum price fluctuation	0.005 = C$5 per contract	0.01 = C$10 per contract
Contract type	Physically-delivered: delivery of eligible Government of Canada bonds	
Last trading day	Trading ceases at 1:00 p.m. on the seventh business day preceding the last business day of the delivery month.	
Expiration day	Expiration occurs on the last trading day	
Delivery notices	Delivery notices must be submitted before 5:30 p.m. or before such time set by the clearing corporation on any business day, between the first business day preceding the first business day of the delivery month, and the first business day preceding the last business day of the delivery month inclusively.	
Delivery day	Delivery shall be made on the first business day following the submission of the delivery notice by the member holding a seller's position or on any other day as determined by the clearing corporation. Delivery shall be completed no later than the last business day of the delivery month.	
Delivery standards	Government of Canada Bonds which: • have a remaining time to maturity of between 1.5 year and 2.5 years; • as of the first day of the delivery month, calculated by rounding down to the nearest whole month period; • have an outstanding amount of at least C$2.4 billion nominal value; • are originally issued at two-year Government of Canada bond auctions; • are issued and delivered on or before the fifteenth day preceding the first delivery notice day of the contract. More information on delivery standards is available in Article 12.112 of the Rules of the Bourse.	Government of Canada Bonds which: • have a remaining time to maturity of between 4.5 years and 5.5 years; • as of the first day of the delivery month, calculated by rounding down to the nearest whole month period; • have an outstanding amount of at least C$3 billion nominal value; • are originally issued at five-year Government of Canada bond auctions; • are issued and delivered on or before the fifteenth day preceding the first delivery notice day of the contract. More information on delivery standards is available in Article 12.212 of the Rules of the Bourse.
Position limit	Information on position limits can be obtained from the Bourse as they are subject to periodic changes. See Circulars. For position limit for the first contract month, please see the First contract month position limit page on the Regulatory Division website.	
Price limit	None	
Minimum margin requirements	Information on minimum margin requirements can be obtained from the Bourse as they are subject to periodic changes. See the Futures contracts margin rates page on the Regulatory Division website.	
Trading hours	Regular session: 8:00* p.m. (t-1) to 4:30 p.m. Note: During early closing days, the regular session closes at 1:30 p.m. *± 15 seconds.	
Clearing corporation	Canadian Derivatives Clearing Corporation (CDCC)	
MX Symbols	CGZ	CGF

	CGB (Ten-Year)	LGB (30-Year)
Underlying	C$100,000 nominal value of a Government of Canada bond with a 6% notional coupon.	C$100,000 nominal value of a Government of Canada bond with a 4% notional coupon.
Expiry cycle	March, June, September and December	
Price quotation	Per C$100 nominal value	
Minimum price fluctuation	0.01 = C$10 per contract	0.05 = C$50 per contract
Contract type	Physically-delivered: delivery of eligible Government of Canada bonds	
Last trading day	Trading ceases at 1:00 p.m. on the seventh business day preceding the last business day of the delivery month.	
Expiration day	Expiration occurs on the last trading day	
Delivery notices	Delivery notices must be submitted before 5:30 p.m. or before such time set by the clearing corporation on any business day, between the first business day preceding the first business day of the delivery month, and the first business day preceding the last business day of the delivery month inclusively.	
Delivery day	Delivery shall be made on the first business day following the submission of the delivery notice by the member holding a seller's position or on any other day as determined by the clearing corporation. Delivery shall be completed no later than the last business day of the delivery month.	
Delivery standards	Government of Canada Bonds which: • have a remaining time to maturity of between 8 years and 10.5 years; • as of the first day of the delivery month, calculated by rounding down to the nearest entire three-month period; • have an outstanding amount of at least C$3.5 billion nominal value; • are originally issued at ten-year Government of Canada bond auctions; • are issued and delivered on or before the fifteenth day preceding the first delivery notice day of the contract. More information on delivery standards is available in Article 12.312 of the Rules of the Bourse.	30Y Government of Canada Bonds benchmark issues only, as determined by the Bourse based on Canadian bond market standards before each roll period and which usually: • have a remaining time to maturity between 29 and 32 years as of the first day of the delivery month; • as of the first day of the delivery month, calculated by rounding down to the nearest entire three-month period; • have an outstanding amount of at least C$12 billion nominal value; • are originally issued at thirty-year Government of Canada bond auctions; • are issued and delivered on or before the fifteenth day preceding the first delivery notice day of the contract. More information on delivery standards is available in Article 12.412 of the Rules of the Bourse.
Position limit	Information on position limits can be obtained from the Bourse as they are subject to periodic changes. See Circulars. For position limit for the first contract month, please see the First contract month position limit page on the Regulatory Division website.	
Price limit	None	
Minimum margin requirements	Information on minimum margin requirements can be obtained from the Bourse as they are subject to periodic changes. See the Futures contracts margin rates page on the Regulatory Division website.	
Trading hours	Regular session: 8:00* p.m. (t-1) to 4:30 p.m. Note: During early closing days, the regular session closes at 1:30 p.m. *± 15 seconds.	Regular session: 7:00* a.m. to 4:30 p.m. Note: During early closing days, the regular session closes at 1:30 p.m. *± 15 seconds.
Clearing corporation	Canadian Derivatives Clearing Corporation (CDCC)	
MX Symbols	CGB	LGB

APPENDIX 2

Options on Government of Canada Bond Futures

	OGZ	OGF	OGB
Underlying	Two-Year Government of Canada Bond Futures	Five-Year Government of Canada Bond Futures	Ten-Year Government of Canada Bond Futures
Trading unit	One Two-Year Government of Canada Bond Futures (OGZ) contract	One Five-Year Government of Canada Bond Futures (OGF) contract	One Ten-Year Government of Canada Bond Futures (OGB) contract
Expiry cycle	Quarterlies: March, June, September and December Serials: Based on the next quarterly futures contract that is nearest to the options contract		
Premium quotation	Quoted in points where each 0.005 point (0.5 basis point) represents C$5		
Minimum fluctuation of the option premium	0.005 = C$5 per contract		
Strike prices	Set at a minimum of 0.10 point interval per Two-Year Government of Canada Bond Futures	Set at a minimum of 0.25 point interval per Five-Year Government of Canada Bond Futures	Set at a minimum of 0.5 point interval per Ten-Year Government of Canada Bond Futures
Contract type	American style		
Last trading day	Trading ceases on the third Friday of the month preceding the options contract month, provided however, that such Friday is a business day and that it precedes, by at least one business day, the first notice day of the underlying futures contract. If it is not a business day, trading will cease on the first preceding business day.		
Exercise	A buyer of one Government of Canada Bond Futures Option may exercise his option on any business day up to and including the last trading day to assume a position in one Government of Canada Bond Futures contract (long, if the option is a call and short, if the option is a put) of a specified contract month at a specified strike price. The seller of one Government of Canada Bond Futures Option incurs, if the option is exercised, the obligation of assuming a position in one Government of Canada Bond Futures contract (short if the option is a call and long if the option is a put) of a specified contract month at a specified strike price.		
Expiration day	Expiration occurs on the last trading day		
Position reporting threshold	250 options or equivalent futures contracts. For the purpose of calculating the reporting limit, positions in the options contracts are aggregated with positions in the underlying futures contracts. For aggregation purposes, the futures equivalent of one in-the-money option contract is one futures contract and the futures equivalent of one out-of-money option contract is half a futures contract.		
Position limit	Information on position limits can be obtained from the Bourse as they are subject to periodic changes. See Circulars.		
Price limit	None		
Minimum margin requirements	Information on minimum margin requirements can be obtained from the Bourse as they are subject to periodic changes.		
Trading hours	Regular session: 8:00* p.m. (t-1) to 4:30 p.m. Note: During early closing days, the regular session closes at 1:30 p.m. *± 15 seconds.		
Clearing corporation	Canadian Derivatives Clearing Corporation (CDCC)		
MX Symbols	OGZ / OGF / OGB		

For more information

Please contact Montréal Exchange if you have any additional questions or require further clarification.

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