



Two-Year Government of Canada Bond Futures (CGZ) Theoretical Model for Pricing the Contract

The Montréal Exchange has reconstituted the theoretical price series of the two-year Government of Canada bond futures contract (CGZ) starting from December 19, 2001 to June 21, 2004. The CGZ bond futures covered over the period include the March 2002 to June 2004 contract months. Pricing is provided on a continuous basis meaning when one futures contract expires, data for the next futures contract is used.

Pricing the CGZ Bond Futures Contract

- Participants of bond futures market know that bond futures prices are linked to the prices of the underlying deliverable bonds. When the bond market rallies, futures prices move up in tandem, and vice versa. An investor needs to identify the "fair price" of a futures contract given a basket of several underlying deliverable bonds prior to designing an effective hedging and trading strategy using bond futures contracts.
- The underlying principle used to identify the fair price of a bond futures contract considers the conditions that would make an investor indifferent between investing in the cash bond and the futures contract.
- The price at which an investor can buy the bond in the cash market and finance the position at the repo financing rate is called the *forward price*. The forward price is essentially equal to the bond price adjusted for the incremental costs (cost of carrying or financing the cash bond purchase) and benefits (coupon interest income) accruing to the buyer of the cash bond.

$$\text{Forward price} = \text{bond price (including accrued interest)} + \text{cost of carry} - \text{coupon income}$$

- Therefore, the theoretical bond futures price is the forward price adjusted for the conversion factor of the deliverable bond.

$$\text{Theoretical bond futures price} = \frac{\text{forward price}}{\text{conversion factor of the bond}}$$

Pricing Model Formula

$$\text{Theoretical bond futures price} = F = \frac{MV + (MV * r * t) - (C * t_c)}{CF}$$

Where:

- F = Theoretical bond futures price
- MV = Market value (spot bond price + accrued interest to settlement date) of deliverable bond
- r = Repo financing rate for the period from settlement to the last delivery day
- t = Term (in years) from settlement date to last delivery day
- t_c = Term (in years) from coupon payment date to last delivery day
- C = Coupon interest income accrued to the bond holder until the last delivery day (per \$100 face value) including any coupons received since settlement and reinvested at the repo financing rate
- CF = Conversion factor for the cheapest-to-deliver bond adjusted for a 6% notional coupon for the CGZ futures contract

➤ Illustrative example

Applying the theoretical pricing model formula for the CGZ futures contract:

DATA as at December 19, 2001 (settlement date on December 21, 2001)

Price of the cheapest-to-deliver bond: CAN 5_% September 1, 2003	103.68
Accrued interest (111 days = September 1 to December 21 settlement date)	1.5966
Market value of bond (full bond price)	105.277
Financing rate (actual repo rate)	2%
Conversion factor of the cheapest-to-deliver bond	0.9894
Days from settlement to futures delivery (December 21 to March 31)	100
Days from coupon payment date to last day of futures delivery (March 1 to March 31)	30

$$\text{Theoretical bond futures price} = F = \frac{MV + (MV * r * t) - (C * t_c)}{CF}$$

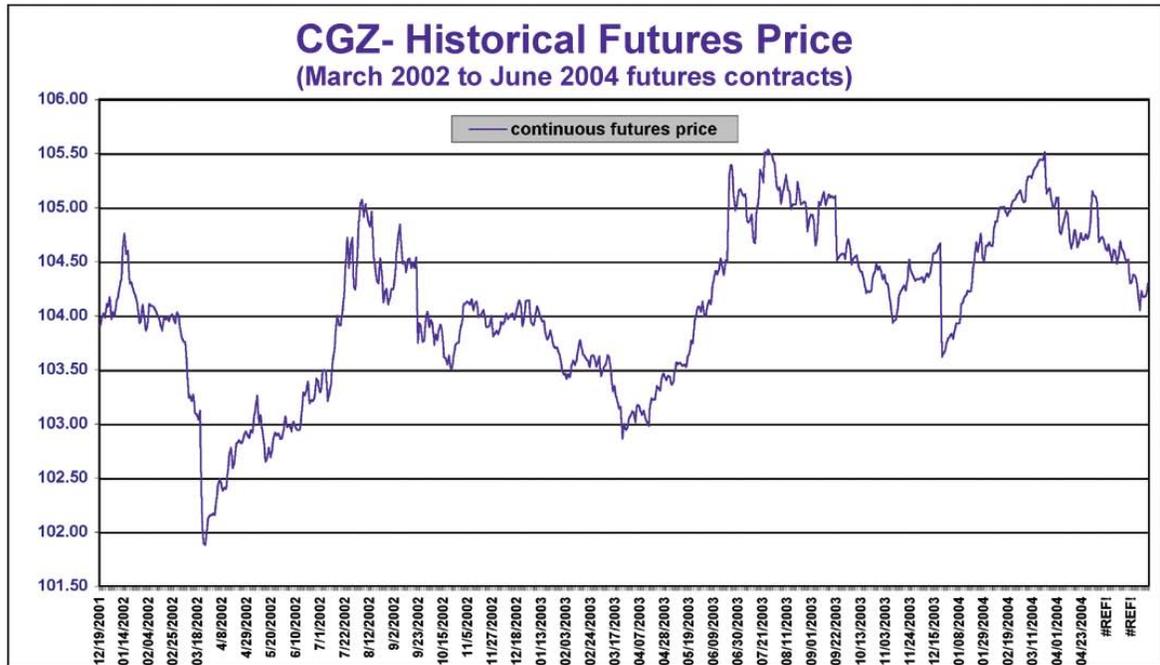
$$F = \frac{105.277 + (105.277 * 0.02 * 100/365) - [(2.625 + (2.625 * 0.02 * 30/365)] + (100 * 0.0525 * 30/365)}{0.9894}$$

$$F = \frac{102.795}{0.9894} = 103.90$$

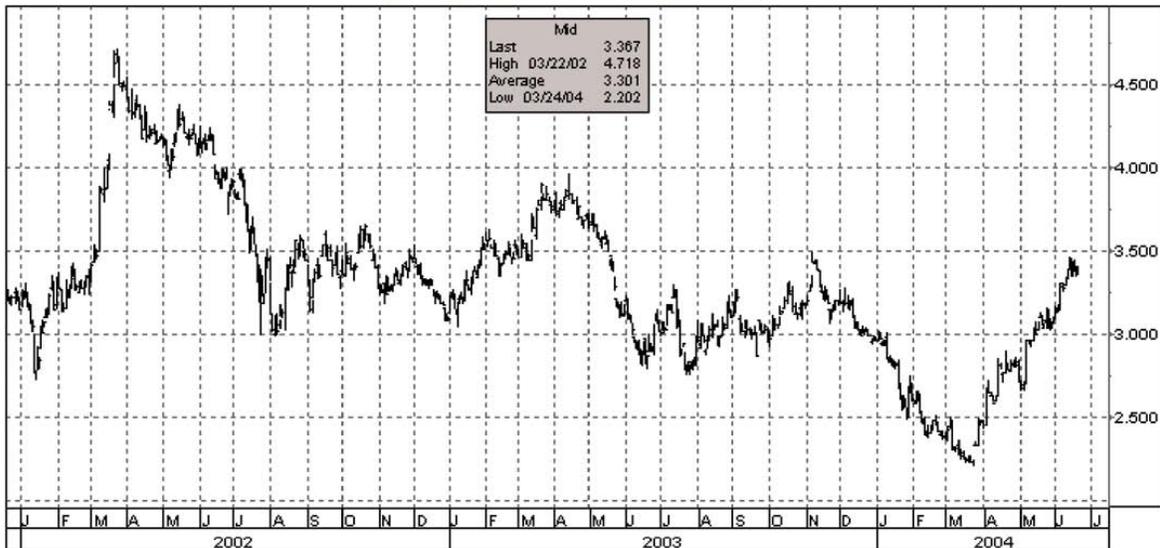
Step-by-step intuitive approach:

STEP BY STEP	AMOUNT (per \$100 face value)	REMARKS
Purchasing the cash bond	103.68 + 1.5966 = \$105.277	Price of bond + accrued interest
Financing costs (cost of carry) until CGZ futures delivery	105.277 * 0.02 * 100/365 = \$0.5769	Amount borrowed to purchase bond * short-term financing rate * number of days/365
Income during the life of the CGZ futures contract (credit and reinvestment of the coupon + accrued interest to the bond holder)	2.625 + (2.625 * 0.02 * 30/365) = \$2.6293 + \$100 * 0.0525 * 30/365 = \$0.4315 Total income = \$2.625 + \$0.4315 = \$3.0608	Coupon income on March 1 + (coupon income * short-term financing rate * number of days/365) + accrued interest to the bond holder from March 1 to March 31 (per \$100 face value)
Forward price of the bond	\$105.277 + \$0.5769 - \$3.0608 = \$102.795	Investment + financing - income
Theoretical March 2002 CGZ futures price (as at December 19, 2001)	102.795/0.9894 = \$103.90	<u>Forward price of bond</u> Conversion factor of bond

APPENDIX



2-year Government of Canada benchmark bond yield (December 2001 to June 2004)



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