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Calculate Fair Value of the CGB (or CGZ, CGF, LGB) Roll

Investors regularly ask us how to calculate the fair value of rolling from the active to new contract for various physical delivery fixed income contracts listed on Montréal Exchange. We discuss here why a complete calculation is unsettlingly complex but suggest that, for most investors, a simplified, option-free, fair value calculation is probably adequate.

Definitions and Theory

Although modestly difficult calculations will occur later in this paper, our initial definition will always remain simple.

The fair value of any contract roll is:

$$FV_{Roll} = FV_{Front} - FV_{Back} \quad (1)$$

Where:

- FV_{Roll} is the fair value of the cost to sell the front (active) contract and buy the back (longer to expiry) contract,
- FV_{Front} is the fair value of the active, shorter time to expiry, contract and,
- FV_{Back} is the fair value of the longer to expiry contract.

Calculating a fair value for the price difference between two contracts, in this methodology, will necessitate calculating the fair value of each of the two contracts and simply taking the price difference.

Readers may recall that combining the sale of a physical delivery futures contract and a purchase of the cheapest-to-deliver bond (CTD) creates an arbitrage to the optimal delivery date of the contract. The fair value, or arbitrage-free value, of a futures contract is thus:

$$FV_{Contract} = \frac{B_0}{CF} - Carry + FV_{Options} \quad (2)$$

Where:

- $FV_{Contract}$ is the fair value of the contract such that the law of one price ensures that no risk-free arbitrage can occur between the cheapest-to-deliver bond and the futures contract.
- B_0 is the (observed) spot price of the cheapest-to-deliver bond.

- CF is the conversion factor for the cheapest-to-deliver bond for this particular contract.
- Carry is the cost-of-carry between the settlement date and the optimal delivery date for the futures contract, and
- $FV_{Options}$ is the computed value of all embedded options in the futures contract, expressed in cents per contract.

As mentioned in the introduction, for Canadian fixed income physical delivery futures, investors can usually safely ignore the embedded options when calculating the fair value of the roll. We discuss our rationale for this argument in detail below but, for now, we express this simplification by dropping the $FV_{Options}$ term from the equation above and simplifying it to:

$$FV_{Contract} = \frac{B_0}{CF} - Carry \quad (3)$$

Where:

- $FV_{Contract}$ is the fair value of the contract where the law of one price ensures that no risk-free arbitrage can occur between the cheapest-to-deliver bond and the futures contract.
- B_0 is the (observed) spot price of the cheapest-to-deliver bond.
- Carry is the cost-of-carry between the settlement date and the optimal delivery date for the futures contract.

Finally, the cost-of-carry for the arbitrage trade of selling the futures contract and buying the cheapest-to-deliver bond is the coupon income on the bond less any financing costs or:

$$Carry = \frac{\left(C + Cr\frac{D_2}{365}\right) - \left(B_0 + AI_0\right) r \frac{D1}{365}}{CF} \quad (4)$$

Where:

- C is any coupon (often zero) received between the settlement and optimal delivery dates,
- r is an observable simple risk-free yield to the optimal delivery date for the contract,
- D_2 is the number of days[1] between the date an interim coupon is received and the delivery date,
- B_0 is the (observed) spot price of the cheapest-to-deliver bond,
- AI_0 is the accrued interest on the settlement date,
- D_1 is the number of days between the settlement date and the optimal delivery date and,
- CF is the conversion factor for the cheapest-to-deliver bond for this particular contract.

In other words, the above calculation calculates the cost of borrowing to buy the bond less any coupon received (and minor income on reinvestment of that coupon) between the date the position is established, and the date delivery is made to end the arbitrage.

Putting the equations 3 and 4 together results in a closed-form equation that gives us the arbitrage-free fair value of the futures contract given the current level of short-term yields, the cheapest-to-deliver bond coupon and price, and the conversion factor for that particular bond; all known quantities[2].

$$FV_{Contract} = \frac{B_0 + AI_0 - AI_2 - C - Cr\frac{D_2}{365} + \left(B_0 + AI_0\right) r \frac{D^1}{365}}{CF} \quad (5)$$

1 Canada standard day count of actual/365 is used here. Adjust appropriately if using this methodology on non-Canadian contracts.

2 There is an incredibly rare possibility that the CTD changes or is likely to change which could affect the inputs to equations 2-5.

Where:

- B_0 is the (observed) spot price of the cheapest-to-deliver bond,
- AI_0 is the accrued interest on the settlement date,
- AI_2 is the accrued interest on the delivery date,
- C is any coupon (often zero) received between the settlement and optimal delivery dates,
- r is an observable simple risk-free yield to the optimal delivery date for the contract,
- D_2 is the number of days[3] between the date an interim coupon is received and the delivery date,
- D_1 is the number of days between the settlement date and the optimal delivery date and,
- CF is the conversion factor for the cheapest-to-deliver bond for this particular contract.

Case Study Example: CGB before the November 2024 Roll

To illustrate the use of equations 1 and 5 above, we present a case study of how an investor could calculate the fair value of the CGB (10-year) roll between the December 2024 and March 2025 contracts on November 25th, 2024.

To calculate the fair value of the December contract, CGBZ24, which has the 2.5% December 2032 Canada bond as the near-certain cheapest-to-deliver, the step-by-step inputs are calculated in Figure 1. In the figure we have mapped the inputs to equation 5 in bold in the rightmost column.

The conversion factor for delivery of that bond into the contract is 0.7802, there was $1.21918 of accrued interest on the settlement date of November 26th, $0.00685 of accrued interest on the expected delivery date of December 2nd, and a coupon of $1.25 on December 1st. From either the overnight index swap market or the prices of Three-Month CORRA (CRA) contracts on Montréal Exchange, we calculated the risk-free rate from settlement to delivery date to be 3.64%.

Using equation 5, above, the fair value of the contract $FV_{Contract}$, for the CGBZ24 contract on this date computes to 121.07, 2 cents higher than the actual closing price of 121.05 on this date. The 2-cent difference reflects the value of all embedded options, which we have ignored in our calculation. In this case the difference represents the value of the wildcard option as the value of all the other options is functionally zero. Valuable options decrease the price of the futures contract as the short position "owns" the options.

FIGURE 1

Trade Specifications

Contract:	CGBZ24
Trade Date:	25-Nov-24
Settlement Date:	26-Nov-24
Delivery Date:	02-Dec-24

3 Canada standard actual/365 is used here, again.

CTD Inputs

CTD Coupon:	2.50%	
CTD Maturity Date:	01-Dec-32	
CTD Conversion Factor:	0.7802	CF
CTD Price Nov25/24:	94.441	B_0
CTD Accrued Nov26/24:	1.21918	AI_0
CTD Accrued Dec2/24:	0.00685	AI_2

Interim Coupon Inputs

Interim Coupon:	1.2500	C
Interim Coupon Date:	01-Dec-24	
Risk Free rate to Delivery:	3.64%	

Daycounts

Settlement to Delivery:	6	D_1
Coupon to Delivery:	1	D_2

Fair Value Ouput

CGBZ24 Fair Value (x-Options)	121.07	$FV_{Contract}$

Historical Close

Actual closing price	121.05

Source: BMO Capital Markets[i] Fixed Income Sapphire database, Montréal Exchange

After we have calculated the fair value of the December contract, we can do the same for the March contract, which we have done below in Figure 2[4]. The fair value of this contract computes to 121.56, 2 cents higher than the observed close, which again reflects the value of embedded options.

FIGURE 2

Trade Specifications

Contract:	CGBH25
Trade Date:	25-Nov-24
Settlement Date:	26-Nov-24
Delivery Date:	03-Mar-25

CTD Inputs

CTD Coupon:	2.75%	
CTD Maturity Date:	01-Jun-33	
CTD Conversion Factor (CF):	0.7909	CF
CTD Price Nov25/24 (B_0):	95.983	B_0
CTD Accrued Nov26/24 (AI1):	1.34110	AI_0
CTD Accrued Mar3/25 (AI2):	0.69315	AI_2

4 Note that, as the 2.50% December 2032 Canada bond has dropped out of the delivery basket for the CGBH25 contract, the CTD becomes the 2.75% June 2033 Canada bond.

Interim Coupon Inputs

Interim Coupon (C):	1.3750	C
Interim Coupon Date:	01-Dec-24	
Risk Free rate to Delivery (r):	3.47%	

Daycounts

Settlement to Delivery (D_1):	97	D_1
Coupon to Delivery (D_2):	92	D_2

Fair Value Ouput

CGBH25 Fair Value (x-Options)	121.56	$FV_{Contract}$

Historical Close

Actual closing price	121.54	

Source: BMO Capital Markets[i] Fixed Income Sapphire database, Montréal Exchange

Finally, we subtract the fair value of the March contract, $121.56, from the fair value of the December contract, $121.07, to arrive at a fair value for the March/December roll (quoted as sell the December, buy the March) of -$0.49. This fair value is identical to the observed value of the cost to roll contracts on November 25th.

FIGURE 3

FV Z24>H25 Roll	
-0.49	FV_{Roll}
Actual Z24>H25 Roll	
-0.49	

Why Can We Ignore The Options?

If the embedded options in the CGB and other contracts cause a 2-cent deviation from the option-free fair value, as shown in Figure 1 and Figure 2, how can we argue that it is reasonable to ignore the value of the options for investors interested in calculating a fair value for the roll between contracts? Our answer lies in the proposal that while the embedded options in some contracts are important, during the roll period, before delivery has become possible on the contracts, the options embedded in the front and back contracts are highly similar and have similar values – often no value at all! Thus, when we take the difference between the fair value of the front contract and the fair value of the back contract, the option values cancel out, and we arrive at a reasonable, although not quite perfect[5], fair value for the roll between the two contracts, as happens in Figure 3.

5 Under most circumstances, the option values will be within a fraction of a cent per contract or even, in many cases, identical at virtually zero value.

Timing Option

The timing option, the option to give notice to deliver anytime between the first and last notice dates, sometimes has value and is important, but is often "decided" well in advance. Cheapest-to-deliver bonds with a coupon rate below that of the overnight rate mean short positions prefer to deliver early (and vice-versa), but outside of emergency rate cuts or hikes by the central bank, there is rarely a quarter when the value of the timing option is not already well known before the roll to a new contract. In the sense that both buyer and seller understand the optimal delivery date in advance, our methodology above, by choosing the optimal delivery date, accounts for the value of the timing option already. Only in surprising situations would our fair value for the roll be incorrect.

Quality (Switch) Option

As we have written many times before, the nature of physical delivery futures contracts in Canada, with conversion factors based on a 6% theoretical coupon for the contracts, a small number of bonds in the delivery basket and old bonds originally issued with longer maturity usually excluded, switch potential in Canada is almost nonexistent. With such a low probability, the quality option becomes almost worthless for both contracts involved in the quarterly roll.

Wildcard Option

The wildcard option often has value in both the CGB (10-year) and CGF (5-year) contracts, but never in the LGB contract and not enough in the CGZ contract to worry about (arguably zero). However, for the CGF and CGB contracts, due to the nature of the wildcard option, which does not begin to decay until the first notice date is reached, the option embedded in both contracts should have very similar values except under these conditions:

1. A (rare) reason to expect volatility to be much higher or lower in 3 months than it is currently or,
2. If there is a very large difference in the coupon for the cheapest-to-deliver bond in the active contract versus the CTD for the next contract.

The first of these scenarios would usually happen when volatility is very elevated today but expected to be much lower in future such as a typical inversion of the volatility surface during times of extreme economic stress. The second situation can happen but is at least as rare since bonds issued just 6 months apart (for the CGB, and even less for the other contracts with wildcard options embedded in them) do not have vastly different coupon rates even when rates are volatile. The combination of these factors means we usually observe wildcard option values that are similar from one contract to the next.

End of Month Option

This embedded option in futures contracts is so obscure that most participants completely ignore it. We concur, especially within the context discussed here, since both contracts would bear an equally unlikely scenario at the end of the month where yields move through the 6% barrier[6] between the last day of trading and the last notice date, resulting in a potential monetization of the end-of-month option. We have never observed even the conditions necessary to monetize this option, never mind the actual monetization by an investor.

Summary

Although more complex calculations can be made to account for the precise, but small, value of the embedded options in futures contracts when calculating the fair value of the roll between contracts, the difference between the more complex computations and this easier method is usually negligible and often zero. The above methodology, if pre-programmed, can easily be linked to a cash bond price feed and measure of the risk-free rate to update the fair value(s) in real-time with virtually no computational lag, unlike a robust calculation/update of wildcard option values.

6 This option can theoretically become modestly valuable when a CTD switch occurs between the final trade of the contract and the delivery of the contract. It requires large changes in interest rates and/or slope of the yield curve.

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