MONTRÉAL EXCHANGE

Index options and correlation trading

Option prices contain important information regarding the market's perception of future risks. The implied probability calculated using option prices across strikes and maturities – referred to as the volatility surface – for a given stock or index, provides relevant insight about the aggregate expectation of market participants about its future volatility.

Interestingly, options on equity indices enclose even more valuable information. Using market prices on index options in combination with option prices of the constituent stocks of the index, one can gain insights into implied correlation, a measure of how the market sees the relative movement of stocks versus each other. A wide range of market participants not only calculate and monitor implied correlation of equity indices but also trade correlation (also referred to as dispersion trading). In this note we will first provide some context and intuition around the concepts of dispersion and correlation, followed by some practical case studies: the S&P/TSX60 and HMMJ CN, a cannabis ETF.

Index correlation and dispersion 101

Before getting into details and specifics of index correlation estimation and dispersion trading, let us develop some intuition around these concepts.

Let us take the case of an index which is composed of 10 stocks of equal weight.

If over a given time period the returns of these 10 stocks are relatively "similar" - in direction and amplitude – then we refer to them as being correlated. Figure 1 shows a sample case of high correlation between the stocks in an index.

FIGURE 1
Stock Returns: High Correlation
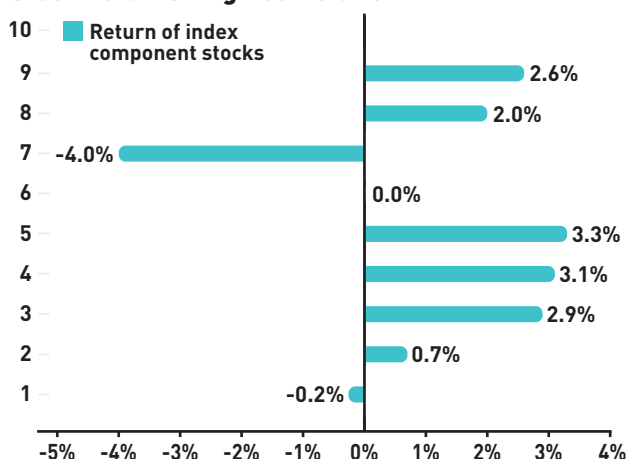
FIGURE 2
Stock Returns: High Dispersion
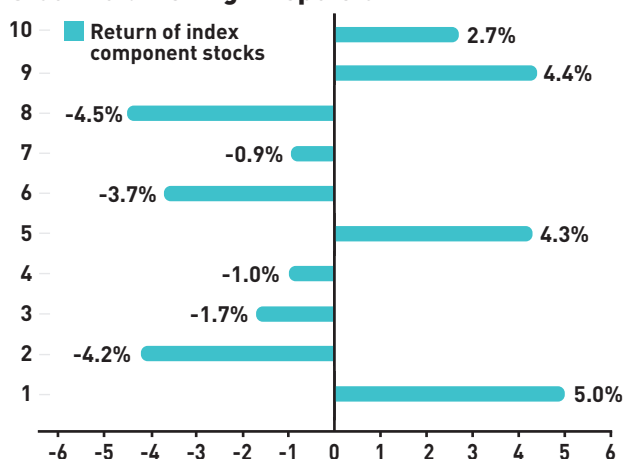
However, if over a given time period the returns of these stocks tend to be randomly distributed among each other, which results in some of them having opposite returns and/or a wide amplitude; then we refer to them as being dispersed. Figure 2 shows a sample case of high dispersion between the stocks in an index.

Naturally, correlation and dispersion are at opposite ends: the higher the correlation between the index components the lower the dispersion and vice versa.

So how do we define and measure index correlation? The most common way to measure index correlation for trading purposes - as we will discuss later - is to look at average pair-wise correlation of stocks in the index.

This means that one measures the correlation between any 2 stocks in the index – over a given time period - across all the possible combinations, and then takes the weighted average.

Let us look at the practical example of Canada's main equity index: the S&P/TSX60 which is composed of 60 stocks. We calculate the correlation of stock 1 vs. 2, then stock 1 vs. 3, 2 vs. 3, etc. over consecutive 3-month periods and then finally make the weighted average – applying the corresponding stock weights - of those numbers. (the general formula is provided in appendix A)

This is called the 3-month realized – because is in the past - correlation of the S&P/TSX60 index as shown in figure 3.

FIGURE 3
Rolling 3-month realized correlation of the S&P/TSX60 since July 2018

Source: Bloomberg

Before we delve into how one can build option strategies to trade correlation/dispersion, let us point out a few stylized facts about index correlation that become handy when looking at creating these trading strategies and evaluating their risks:

- When equity markets are under selling pressure, correlation among stocks goes up, therefore index realized correlation picks up. This can be explained by the fact that in a down market, the price of a majority of stocks, if not all of them tends to fall. In a down market sellers are less price sensitive and what matters most is reducing risk by selling stocks.

- Conversely, (realized) correlation tends to be lower in periods of "calm" in the market. Because, during these times, stock picking is more present while macro factors and sentiment take a back seat.

- This means that correlation – both implied and realized - increases when markets are volatile.

- Finally, by definition (realized) correlation can not exceed 100%.

Figure 4 shows the 3-month realized correlation of the S&P/TSX60 stocks vs. the S&P/TSX60 price during 2018.

Prior to the market sell-off in February 2018 the inter-stock correlation (blue line) dropped to lows of 10%. Once the market sold off in February tensions developed through April correlation increased to 40%. The following period of calm it dropped back to 20% in the summer as stock performance dispersed. The market turmoil that culminated in the sell-off near the year-end saw inter-stock correlation raise towards 40% again as investors sold more indiscriminately.

FIGURE 4
Rolling 3-month realized correlation vs. index price level for the S&P/TSX60 (2018)

Source: Bloomberg

Now that we are well aware of the close relationship between volatility and correlation, the questions that come to mind are:

a. How do we define a measure of implied correlation (a comparable concept to implied volatility)?

b. How do we trade correlation (a comparable concept to trading volatility)?

We will answer these questions, first by defining and discussing implied correlation before looking into the practical cases of the S&P/TSX60 and HMMJ CN.

Implied correlation 101

Once again, we will leave the technical details of how to calculate implied correlation using options prices (see appendix A) aside to focus on the intuition around what implied correlation is and how it can be traded.

The sample case of an index (let us call it IDX) with only 2 constituents (stock A and stock B) with 50% weight for each in the index, will provide us with a very practical and easy way to grasp this topic.
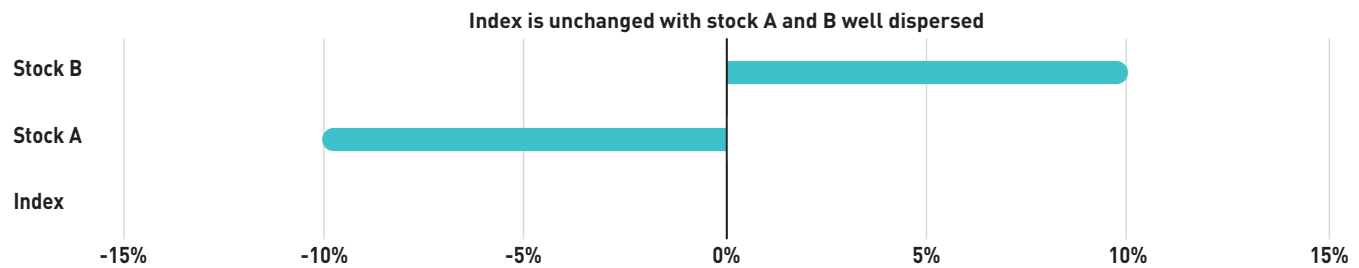
Case 1: high dispersion

Let's take a case where over a 1-month period stock A drops 10% and stock B rallies 10%.

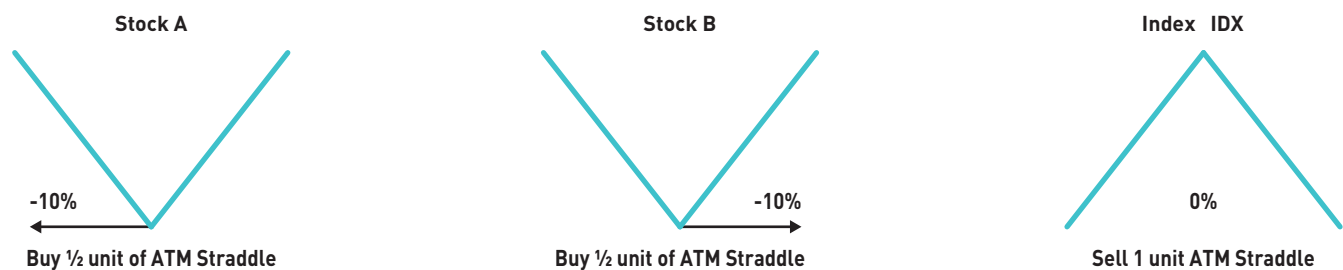What happens to the index (IDX) during this period? Answer: it ends up unchanged.

FIGURE 5
Example of 3-month returns of stock A and B result in a 0% index return for the period

Index is unchanged with stock A and B well dispersed

Now let us consider the following sample trade. At the beginning of the period, the trader:

- Buys 0.5 notional units of at-the-money straddle on stock A for 5%, and
- Buys 0.5 notional units of at-the-money straddle on stock B for 12%, and
- Sells 1 notional unit of at-the-money straddle on the index IDX for 6%

Stock A	Stock B	Index IDX
-10%	-10%	0%
Buy ½ unit of ATM Straddle	Buy ½ unit of ATM Straddle	Sell 1 unit ATM Straddle

What is the P&L of this trade at the end of the period?

0.5 * (Stock A move of 10% - cost of 5% of the straddle A)

+ 0.5 * (Stock B move of 10% - cost of 12% cost of straddle B)

− (Index move of 0% - cost of 6% straddle IDX) = **+7.5%**

What happened? The fact that the stocks returns are dispersed allows the trader to collect the premium of the short straddle position on the index (which does not move due to stock dispersion), while the stocks having moved (up and down) during the period pay part or the entirety of the premium spent on the straddles he/she purchased.

Case 2: high correlation

Now let us take a case where the price of the straddles on the stock and index are the same, but both stock A and B rally by 10% during the period.

What happens to the index (IDX) during this period? Answer: it rallies by 10% , the same amount of the stocks.

FIGURE 6
Example of 3-month returns of stock A and B result in a 10% index return for the period

Index return with stock A and B fully correclated
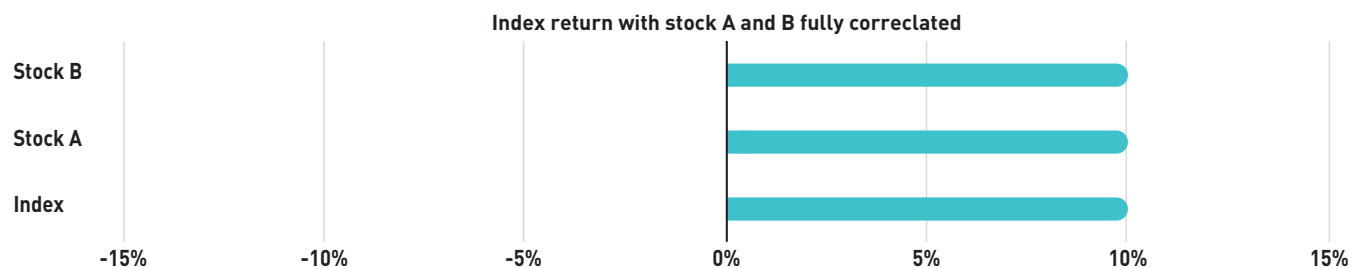
We will do the same trade, sell the index (IDX) straddle to buy ½ unit each of straddles on stock A and B. The only thing that has changed compared to the pervious example is the return on stock A - now fully correlated with stock B - and as a result the return on the index, is now 10% instead of 0%.
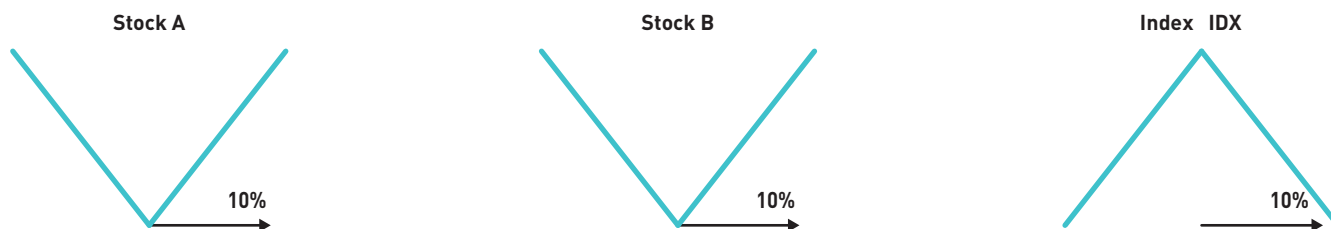
Stock A	Stock B	Index IDX
10%	10%	10%

What is the P&L of this trade at the end of the period?

$$0.5 * (\text{Stock A move of 10\% - cost of 5\% of the straddle A})$$

$$+ 0.5 * (\text{Stock B move of 10\% - cost of 12\% cost of straddle B})$$

$$- (\text{Index move of 10\% - cost of 6\% straddle IDX}) = \textbf{-3.5\%}$$

What happened? The fact that the stocks returns are correlated results in an equally important move in the index. The loss on the short straddle position in the index superseded the overall gains on long straddle positions on stocks.

Main take away: Implied correlation is a measure of how expensive (or cheap) options on an index are relative to a basket of options of its constituent stocks.

Here a few salient facts that capture the essence of correlation trading and implied correlation:

a. Everything else equal (i.e. straddle prices and the absolute level of change in stock prices), the P&L of selling an index straddle versus buying the basket of straddles on constituent stocks, depends directly on the level of actual (i.e. realized) correlation between the constituent stocks.

b. Using options prices for an index and its constituent stocks for a given maturity, one can calculate the implied correlation for that maturity (appendix A).

c. Implied correlation can be viewed as the market's projection of how correlated the index constituents will be in the future.

d. If one thinks that the level of implied correlation is too high, he/she can sell index options and buy the weighted basket of its constituents: this is referred to as selling correlation.

Now, we will discuss the practical aspects of correlation trading and provide a few concrete examples.

Trading correlation

Correlation trades have been around since options started trading. Traditionally many option traders and in particular market makers build portfolios of options that end up being long single stock options and short index options. As such, these institutional participants have often a short correlation exposure.

To generate profits, one needs to sell high implied correlation, with the view that the subsequent correlation between stocks will be lower than the level he/she sold it. The higher the implied correlation sold the higher the probability that the trade will make profits.

In fact, if one could sell an implied correlation of 100% - which is almost never reached in practice - then he/she is guaranteed not to incur any losses and has an extremely high likelihood of making a profit. This is because the subsequent inter-stock realized correlation is capped at 100% by definition, so the trade will have a positive P&L.

The case of the S&P/TSX60 index

In practice implied correlation on most indices seldom reaches levels above 70%. Figure 7 shows the 3-month implied correlation of the S&P/TSX60 since June 1st, 2018.

Rolling 3-month implied correlation of the S&P/TSX60 (since June 2018)
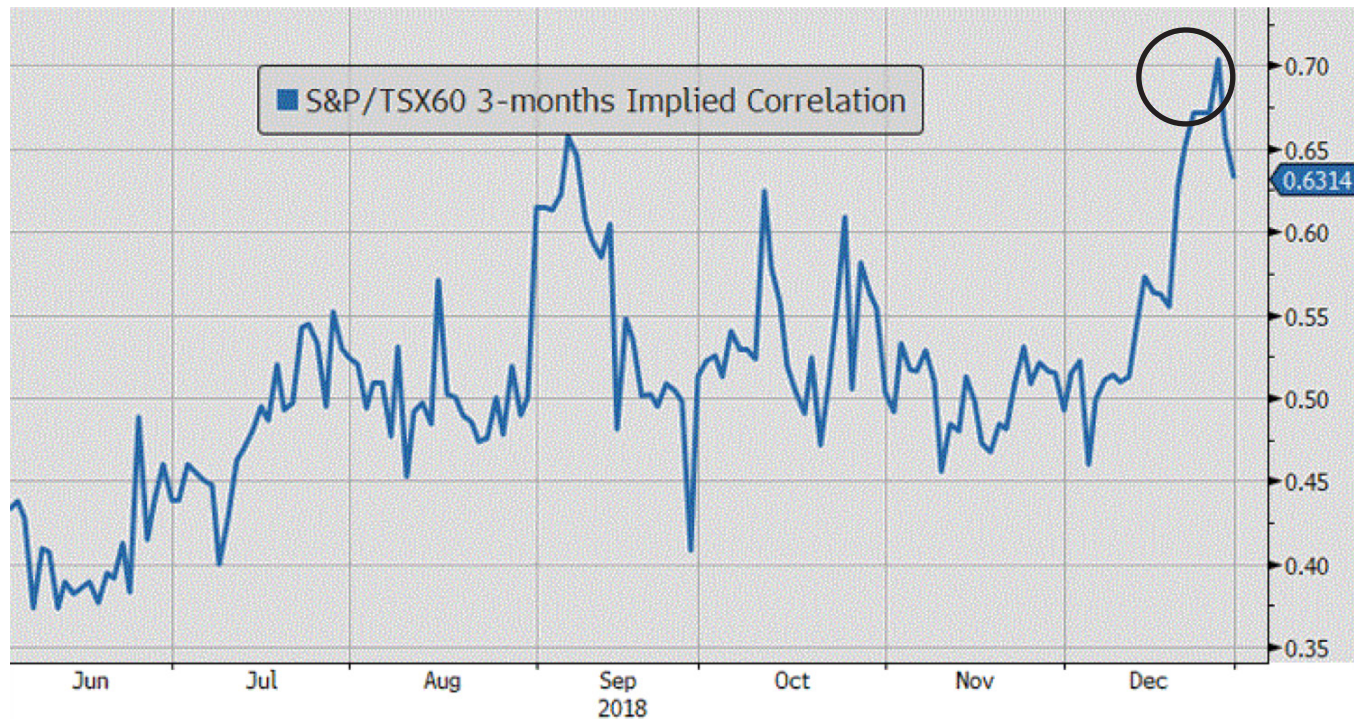
Source: Bloomberg

Before the financial crisis of 2008, many traders would sell (implied) correlation at levels sometimes as low as 25-30%. However, post-crisis the general level of correlation among stocks appears to have reset higher, this makes selling correlation at those levels less compelling. Presently, once implied correlation exceeds 40-50% on North American major equity indices, it attracts the attention of many institutional participants.

To show a practical example of how a correlation trade can works in practice, we will describe here how one could have sold 3-months implied correlation on S&P/TSX60 on September 6th, 2018 when it reached 64%. (This sample trade is for illustration purpose sonly and does not constitute a trade recommendation or investment advice. Please refer to appendix B for details on assumptions for this backtest)

The trade is initiated by:

1. Selling $100 notional of Dec 2018 (3 month to expiry) at-the-money straddle on the S&P/TSX60. These options are listed on the Montreal Exchange and carry the SXO symbol.

2. Buying Dec 2018 at-the-money straddles on the top (by weight) 14 constituents of the index.

   Why trade only 14 constituents and not all 60 of them?

   It is common for traders to limit themselves to the basket of index constituents with highest weights, simply because these stocks usually tend to be more liquid and the options on them are also liquid as a result. For constituents with lower weightings, the bid/ask spread of option prices can be prohibitively high to such extent that it overpowers the tracking error that one would be subject to by not including the options on these stocks in the trade.

   In the present example these 14 stocks represent 61% of the weight of the index. We prorate the remaining 39% weight among these stocks so that the total weight becomes 100%.

   Note: the present example uses mid-market prices on straddles, however, in reality one has to cross the bid/ask spread which can be costly and must be accounted for before initiating a trade. Figures are net of transaction commissions (i.e. they are reflected in the prices) which represent around 0.60% of the notional for the overall basket trade.

TABLE 1
Specifics of a sample trade initiated on Sep 6th, 2018 and closed on Dec 20th 2018

Action	Underlying	Weight in index (%)	Adjusted Weight	Notional per Trade	Opening Straddle price*	Closing Straddle price	Profit/Loss in $
SELL	S&P/TSX 60 INDEX	100%	100%	**$-100.00**	5.13%	11.5%	**-$6.37**
BUY	ROYAL BANK OF CANADA	8.9%	14.6%	$14.62	5.02%	12.3%	$1.06
BUY	TORONTO-DOMINION BANK	8.3%	13.7%	$13.70	5.23%	16.1%	$1.49
BUY	BANK OF NOVA SCOTIA	5.5%	9.0%	$8.99	5.37%	9.0%	$0.33
BUY	CANADIAN NATL RAILWAY CO	5.1%	8.4%	$8.38	6.87%	13.4%	$0.55
BUY	ENBRIDGE INC	5.0%	8.2%	$8.24	8.50%	5.0%	**-$0.28**
BUY	SUNCOR ENERGY INC	4.3%	7.1%	$7.06	8.98%	28.7%	$1.40
BUY	BANK OF MONTREAL	3.9%	6.3%	$6.34	4.95%	19.5%	$0.92
BUY	BCE INC	3.1%	5.2%	$5.18	5.65%	2.3%	**-$0.17**
BUY	TRANSCANADA CORP	3.0%	5.0%	$5.00	6.57%	9.7%	$0.16
BUY	CAN IMPERIAL BK OF COMMERCE	3.0%	5.0%	$4.95	5.02%	17.4%	$0.61
BUY	BROOKFIELD ASSET MANAGE-CL A	3.0%	4.9%	$4.92	4.80%	8.0%	$0.16
BUY	NUTRIEN LTD	2.7%	4.4%	$4.45	6.58%	15.9%	$0.41
BUY	MANULIFE FINANCIAL CORP	2.6%	4.2%	$4.22	7.48%	15.8%	$0.35
BUY	CANADIAN PACIFIC RAILWAY	2.4%	4.0%	$3.95	6.60%	12.9%	$0.25
	Total for Basket of constituents	60.7%	100.0%				$7.23
	Total trade P&L						**$0.85**

Source: Bloomberg, LFC calculations

Table 1 shows that the trade resulted in a hypothetical profit of 0.85%. If the S&P/TSX60 straddle was worth more (than the 5.3% it was sold it at) at the initiation of the trade – which means that the implied correlation was higher – one can see that the overall trade would have been more profitable.

It is very important to note that the presented trade is only one of many ways one can implement a correlation trade. For instance, in this example, straddle positions were not delta hedged over the life of the trade. In practice, traders often delta hedge their positions.

Furthermore, the P&L of an ideal or "perfect" correlation trade should solely depend on the realized pair-wise correlation of the stocks over the life of the trade. But, the use of straddles creates path-dependency. Why? the same level of realized correlation can be achieved with stocks having different price patterns between the start and the end of the trade. This means the use of straddles can result in different P&L for the same final (realized) correlation levels depending of which path stocks took. We will not discuss further the intricacies of correlation trading, suffice to say that what is presented here is one way, though simplified, in which to try and capture the difference between implied and realized correlation.

Takeaway: for a correlation trade (selling) to have the potential to be profitable, one needs to identify situations where index (or ETF) options are "rich" in relative terms versus the options on its constituent stocks AND where one expects that constituent stocks will have a low correlation (i.e. disperse) among themselves going forward. This is usually more likely when there are many stock-specific risks and/or news expected in the future.

The case of the HMMJ CN

Cannabis stocks and ETFs, exhibit high volatility as discussed in our previous article "**High volatility: the case of cannabis stocks**". Additionally, as the industry is still in its growth and development stage, cannabis stocks have a high level of embedded stock-specific and idiosyncratic risk.

Elevated ETF option premia combined with a high potential for stock-specific news and stories make the cannabis sector an interesting area to look for potential correlation trades. If implied correlation reaches high levels then

selling correlation might present interesting risk-reward.

To test our institution, we calculated the implied correlation of HMMJ CN (Horizon Marijuana Life Sciences ETF).

There are a number of practical approximations we needed to undertake for our calculations. First, some of the constituent stocks of HMMJ CN do not have listed options, we therefore choose the stocks with the highest weight and available options. This resulted in a list of 7 stocks which make up 68% of HMMJ CN's weight. A basket of these stocks with pro-rated weights (to make a total of 100%) tracks quite well the ETF's price performance.

Second, 3 of these 7 stocks have only US listed options. For the purpose of our calculation we assumed that the implied volatility of a $CAD denominated option would be the same as the $USD denominated options. In other words, we assume that the effect of the currency (i.e. USD vs. CAD exchange rate) volatility on overall option pricing is negligible, and that the currency and stock prices are uncorrelated. (This sample trade is for illustration purposes only and does not constitute a trade recommendation or investment advice. Please refer to appendix B for details on assumptions for this backtest)

FIGURE 8
Rolling 3-month implied correlation of the HMMJ CN ETF (since June 2018)
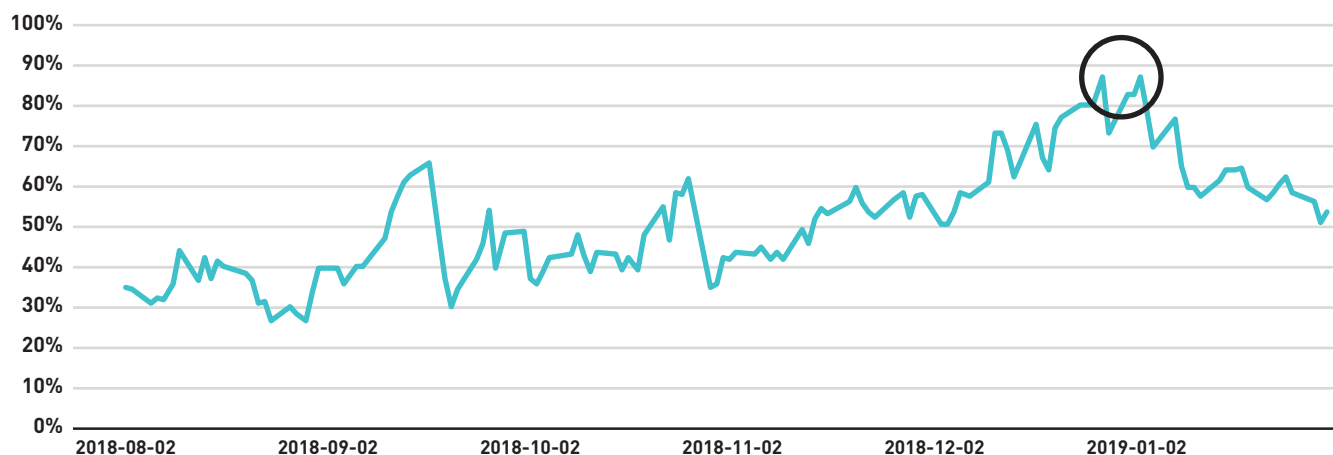
Source: Bloomberg, LFC calculations

The results are very interesting. As figure 8 shows, implied correlation in the cannabis sector is relatively high in general. This is not surprising. Generally speaking, stocks in this sector are correlated and tend to move in tandem in both good and bad days for the sector. However, in December 2018 the implied correlation reached almost a staggering 90%!

The risk reward of selling implied correlation at 90% can be quite interesting, given that subsequent realized correlation can not exceed 100% in any case.

Additionally, while these stocks exhibit high level of (realized) correlation, there remains plenty of stock specific news and events which can be good catalysts for stock dispersion. In particular the sector is undergoing a wide range of consolidation, mergers and acquisitions that create idiosyncratic risks.

A trade to sell 3-month implied correlation would have been initiated on December 27th, 2018 by:

1. Selling $100 notional of Mar 2019 (3 month to expiry) at-the-money straddle (14 strike) on the HMMJ CN. These options are listed on the Montreal Exchange.

2. Buying Mar 2019 at-the-money straddles on the top 7 constituents of the index.

In the present example these 7 stocks represent 68% of the weight of the index. We prorate the remaining 32% among these stocks so that he total weight becomes 100%.

The present example uses mid-market prices on straddles, however, in reality one has to cross the bid/ask spread which can be costly and must be accounted for before initiating a trade. Figures are net of transaction commissions (i.e. they are reflected in the prices) which represent around 0.30% of the notional.

TABLE 2
Specifics of a sample trade initiated on Sep 6th, 2018 and as of Feb 1st, 2019

Action	Underlying	Weight in index (%)	Adjusted Weight	Notional per Trade	Opening Straddle price*	Closing Straddle price	Profit/Loss in $
SELL	HMMJ CN Equity	100%	100%	**$-100.00**	26.21%	47.4%	**-$21.21**
BUY	APHA CN Equity	6.8%	10.0%	$9.95	38.63%	47.0%	$0.83
BUY	SMG US Equity	8.2%	12.0%	$12.02	14.23%	24.4%	$1.23
BUY	TLRY US Equity	8.4%	12.3%	$12.29	39.84%	28.5%	**-$1.40**
BUY	WEED CN Equity	13.6%	20.0%	$19.98	27.78%	79.3%	$10.30
BUY	ACB CN Equity	11.1%	16.4%	$16.36	31.00%	38.0%	$1.14
BUY	CRON CN Equity	10.7%	15.7%	$15.65	28.29%	75.5%	$7.39
BUY	GWPH US Equity	9.4%	13.7%	$13.74	23.64%	51.4%	$3.82
	Total for Basket of constituents	68.1%	100.0%	$100.00			$23.32
	Total trade P&L						**$2.10**

As table 2 shows, 1 month into the life of the trade - with nearly two months to expiry - on January 30th the trade has generated 4.37% profit. The main drivers behind this P&L are the strong outperformance of CRON CN and WEED CN which nearly doubled in price in a month. Even though all constituents rallied over this period, there was a wide range of stock performance. This shows that dispersion does not necessarily mean performance in opposite direction, but rather a wide range for stock returns.

While our intuition about HMMJ CN was validated, the reader must keep in mind that trade slippage is likely to take away an important part of profits in practice. The implementation of this type of trade also presents execution risk as there is a need for trading multiple options simultaneously.

Tracking both implied and realized correlation on equity indices is a useful tool in gauging a variety of market and option related matters such as: the richness of equity indices options versus single stock or market participants' perception of future correlation among stocks. It also has practical applications in risk management via the variance/covariance matrices which are part of the behind the scenes of most portfolio risk systems and methodologies. Lastly, the wide range of ETF and indices with liquid options provide the initiated and institutional investors with a viable source of returns should one take a systematic approach in identifying the opportunities.

Appendix A

Calculating implied correlation of an index

The variance of an index I with N constituent stocks can be written as:

$$\sigma_I^2 = \sum_{i=1}^{N} w_I^2 \, \sigma_I^2 + 2 \sum_{i=1}^{N-1} \sum_{J>1}^{N} w_i \, w_i \, \sigma_i \, \sigma_j \, \rho_{ij}$$

Where σ_i and σ_j are the volatility of the ith and jth index constituents, w_i and w_i are the weight of the ith and jth index constituents. ρ_{ij} is the pair-wise correlation of index components.

We can solve this equation in order to isolate the average stock correlation in the index. The implied correlation is therefore:

$$\rho_I = \frac{\sigma_I^2 - \sum_{i=1}^{N} w_I^2 \, \sigma_I^2}{2 \sum_{i=1}^{N-1} \sum_{J>1}^{N} w_i \, w_i \, \sigma_i \, \sigma_j \, \rho_{ij}}$$

Appendix B

Assumptions for backtests

S&PTX60 example

- Opening trade date Sep 6th, 2018
- Closing trade date Dec 20th, 2018
- $1 commission per option per side
- Option prices are end of day mid-market prices: source Bloomberg
- On Sep 6th, options for the December 2018 expiry were not listed on BAM/A CN, NTR CN et CP CN, the opening prices were calculated using estimated mid-market implied volatility for the relevant maturity based on an extrapolation from the neighboring listed maturities
- 14 stocks from the index constituents were only included in the trade
- Stocks weights were increased on a pro-rate basis in line with their original weight in the index so that the total weight of the basket of 14 stocks is equal to 100%.

HMMJ CN example

- Opening trade date Sep 6th, 2018
- Date on which the P&L is calculated January 31st, 2019
- $1 commission per option per side
- Option prices are end of day mid-market prices: source Bloomberg
- On Dec 27th, options for the March 2019 expiry were not listed for CRON CN and GWPH US. The opening prices were calculated using estimated mid-market implied volatility for the relevant maturity based on an extrapolation from the neighboring listed maturities
- On Jan 31st, options for the March 2019 expiry were not listed on CRON CN, the end of day mid-market prices were calculated using estimated mid-market implied volatility for the relevant maturity based on an extrapolation from the neighboring listed maturities
- 7 stocks from the ETF constituents were only included in the trade
- Stocks weights were increased on a pro-rate basis in line with their original weight in the ETF so that the total weight of the basket of 7 stocks is equal to 100%.

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